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About the Claimed 'Longitudity' of the Antisymmetric Tensor Field After Quantization*

Valeri V. Dvoeglazov†
Escuela de Física, Universidad Autónoma de Zacatecas
Antonio Dovalí Jaime s/n, Zacatecas 98068, ZAC., México
Internet address: VALERI@CANTERA.REDUAZ.MX

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Abstract

It has long been claimed that antisymmetric tensor field of the second rank is longitudinal after quantization. Such a situation is quite unacceptable from a viewpoint of the Correspondence Principle. On the basis of the Lagrangian formalism we calculate the Pauli-Lyuban'sky vector of relativistic spin for this field. Even at the classical level it can be equal to zero after application of the well-known constraints. The correct quantization procedure permits us to propose solution of this puzzle in the modern field theory. Obtained results develop the previous consideration of Evans [*Physica A*214 (1995) 605-618].

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†On leave of absence from *Dept. Theor. & Nucl. Phys., Saratov State University, Astrakhanskaya ul., 83, Saratov RUSSIA.* Internet address: dvoeglazov@main1.jinr.dubna.su

Quantum electrodynamics (QED) is a construct which found overwhelming experimental confirmations (for recent reviews see, *e.g.*, refs. [1,2]). Nevertheless, a number of theoretical aspects of this theory deserves more attention. First of all, they are: the problem of "fictious photons of helicity other than $\pm j$, as well as the indefinite metric that must accompany them"; the renormalization idea, which "would be sensible only if it was applied with finite renormalization factors, not infinite ones (one is not allowed to neglect [and to subtract] infinitely large quantities)"; contradictions with the Weinberg theorem "that no symmetric tensor field of rank j can be constructed from the creation and annihilation operators of massless particles of spin j", *etc.* They were shown at by Dirac [3,4] and by Weinberg [5]. Moreover, it appears now that we do not yet understand many specific peculiarities of classical electromagnetism, first of all, the problems of longitudinal modes and of the Coulomb action-at-a-distance, refs. [6–13]. Secondly, the standard model, which has been constructed on the base of ideas, which are similar to QED, appears to be no able to explain many puzzles in neutrino physics.

In my opinion, all these shortcomings are the consequences of ignoring several important questions. "In the classical electrodynamics of charged particles, a knowledge of $F^{\mu\nu}$ completely determines the properties of the system. A knowledge of A^{μ} is redundant there, because it is determined only up to gauge transformations, which do not affect $F^{\mu\nu}$... Such is not the case in quantum theory..." [14]. We learnt, indeed, about this fact from the Aharonov-Bohm [15] and the Aharonov-Casher effects [16]. However, recently several attempts have been undertaken to explain the Aharonov-Bohm effect classically [17]. These attempts have, in my opinion, logical basis and are in complete accordance with the Correspondence Principle. In the mean time, quantization of the antisymmetric tensor field led us to a new puzzle, which until now was not drawn much attention to. It was claimed that the antisymmetric tensor field of the second rank is longitudinal after quantization [18–22]. We know that the antisymmetric tensor field (electric and magnetic fields, indeed) is transversal in the Maxwellian classical electrodynamics. It is clear that longitudinal components can not be transformed into the transversal ones in the $\hbar \to 0$ limit.[1] How should we manage with the Correspondence Principle in this case? It is often concluded: one is not allowed to use the antisymmetric tensor field to represent the quantized electromagnetic field in relativistic quantum mechanics. Nevertheless, we are convinced that a reliable theory should be constructed on the base of a minimal number of ingredients ("Occam's Razor") and should have well-defined classical limit. Therefore, in this paper we undertake a detailed analysis of rotational properties of the antisymmetric tensor field, we calculate the Pauli-Lyuban'sky vector of relativistic spin (which defines, what the field is: transversal or longitudinal) and we then conclude, whether it is possible to obtain conventional electromagnetic theory with transversal modes provided that strengths (*not* potentials) are chosen to be physical variables. The particular case also exists when the Pauli-Lyuban'sky vector for the antisymmetric tensor field of the second rank is equal to zero, what corresponds to the claimed longitudity (helicity $h = 0$) of this field.

Researches in this area from a viewpoint of the Weinberg's $2(2j+1)$ component theory have been started in refs. [26–30,9–12]. I would also like to point out that the problem at hand is directly connected with our understanding of the nature of neutral particles,

[1] See also group-theoretical consideration in ref. [23] which concerns with the reduction of transversal rotational degrees of freedom to gauge degrees of freedom in infinite-momentum/zero-mass limit. The only mentions of the transversality of the quantized antisymmetric tensor field see in refs. [24,25].

including neutrinos [31–38]. From a mathematical viewpoint theoretical content does not depend, what representation space, which field operators transform on, has been chosen.

We begin with the antisymmetric tensor field operator (in general, complex-valued):

$$F^{\mu\nu}(x) = \sum_{\eta} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_p} \left[F^{\mu\nu}_{\eta\,(+)}(\mathbf{p})\, a_\eta(\mathbf{p})\, e^{-ip\cdot x} + F^{\mu\nu}_{\eta\,(-)}(\mathbf{p})\, b_\eta^\dagger(\mathbf{p})\, e^{+ip\cdot x}\right] \tag{1}$$

and with the Lagrangian, including, in general, mass term:[2]

$$\mathcal{L} = \frac{1}{4}(\partial_\mu F_{\nu\alpha})(\partial^\mu F^{\nu\alpha}) - \frac{1}{2}(\partial_\mu F^{\mu\alpha})(\partial^\nu F_{\nu\alpha}) - \frac{1}{2}(\partial_\mu F_{\nu\alpha})(\partial^\nu F^{\mu\alpha}) + \frac{1}{4}m^2 F_{\mu\nu}F^{\mu\nu} \quad . \tag{3}$$

The Lagrangian leads to the equation of motion in the following form:

$$\frac{1}{2}(\square + m^2)F_{\mu\nu} + (\partial_\mu F_{\alpha\nu}{}^{,\alpha} - \partial_\nu F_{\alpha\mu}{}^{,\alpha}) = 0 \quad , \tag{4}$$

where $\square = -\partial_\alpha \partial^\alpha$. It is this equation for antisymmetric-tensor-field components that follows from the Proca-Bargmann-Wigner consideration:

$$\partial_\alpha F^{\alpha\mu} + \frac{m}{2} A^\mu = 0 \quad , \tag{5}$$

$$2mF^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad , \tag{6}$$

provided that $m \neq 0$ and in the final expression we take into account the Klein-Gordon equation $(\square - m^2)F_{\mu\nu} = 0$. The latter expresses relativistic dispersion relations $E^2 - \mathbf{p}^2 = m^2$ and it follows from the coordinate Lorentz transformation laws [39, §2.3].

Following the variation procedure given, *e.g.*, in refs. [40–42] one can obtain that for rotations $x^{\mu'} = x^\mu + \omega^{\mu\nu}x_\nu$ the corresponding variation of the wave function is found from the formula:

$$\delta F^{\alpha\beta} = \frac{1}{2}\omega^{\kappa\tau} \mathcal{T}^{\alpha\beta,\mu\nu}_{\kappa\tau} F_{\mu\nu} \quad . \tag{7}$$

The generators of infinitesimal transformations are then defined as

$$\begin{aligned} \mathcal{T}^{\alpha\beta,\mu\nu}_{\kappa\tau} &= \frac{1}{2}g^{\alpha\mu}(\delta^\beta_\kappa\,\delta^\nu_\tau - \delta^\beta_\tau\,\delta^\nu_\kappa) + \frac{1}{2}g^{\beta\mu}(\delta^\nu_\kappa\delta^\alpha_\tau - \delta^\nu_\tau\,\delta^\alpha_\kappa) + \\ &+ \frac{1}{2}g^{\alpha\nu}(\delta^\mu_\kappa\,\delta^\beta_\tau - \delta^\mu_\tau\,\delta^\beta_\kappa) + \frac{1}{2}g^{\beta\nu}(\delta^\alpha_\kappa\,\delta^\mu_\tau - \delta^\alpha_\tau\,\delta^\mu_\kappa) \quad . \end{aligned} \tag{8}$$

It is $\mathcal{T}^{\alpha\beta,\mu\nu}_{\kappa\tau}$, the generators of infinitesimal transformations, that enter in the formula for the relativistic spin tensor:

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[2] The massless limit ($m \to 0$) of the Lagrangian is connected with the Lagrangians used in the conformal field theory and in the conformal supergravity by adding the total derivative:

$$\mathcal{L}_{CFT} = \mathcal{L} + \frac{1}{2}\partial_\mu \left(F_{\nu\alpha}\partial^\nu F^{\mu\alpha} - F^{\mu\alpha}\partial^\nu F_{\nu\alpha}\right) \quad . \tag{2}$$

The gauge-invariant form [18] is obtained only if take into account the generalized Lorentz condition, see ref. [19] and what is below.

$$J_{\kappa\tau} = \int d^3\mathbf{x} \left[\frac{\partial \mathcal{L}}{\partial(\partial F^{\alpha\beta}/\partial t)} \mathcal{T}_{\kappa\tau}^{\alpha\beta,\mu\nu} F_{\mu\nu} \right] \quad . \tag{9}$$

As a result we obtain:

$$\begin{aligned} J_{\kappa\tau} = \int d^3\mathbf{x} \, [(\partial_\mu F^{\mu\nu})(g_{0\kappa} F_{\nu\tau} - g_{0\tau} F_{\nu\kappa}) - (\partial_\mu F^{\mu}{}_{\kappa}) F_{0\tau} + (\partial_\mu F^{\mu}{}_{\tau}) F_{0\kappa} + \\ + F^{\mu}{}_{\kappa} (\partial_0 F_{\tau\mu} + \partial_\mu F_{0\tau} + \partial_\tau F_{\mu 0}) - F^{\mu}{}_{\tau} (\partial_0 F_{\kappa\mu} + \partial_\mu F_{0\kappa} + \partial_\kappa F_{\mu 0})] \quad . \end{aligned} \tag{10}$$

If agree that the orbital part of the angular momentum

$$L_{\kappa\tau} = x_\kappa \Theta_{0\,\tau} - x_\tau \Theta_{0\,\kappa} \quad , \tag{11}$$

with $\Theta_{\tau\lambda}$ being the energy-momentum tensor, does not contribute to the Pauli-Lyuban'sky operator when acting on the one-particle free states (like the Dirac $j = 1/2$ case), then the Pauli-Lyuban'sky 4-vector is constructed as follows [43, Eq.(2-21)]

$$W_\mu = -\frac{1}{2} \epsilon_{\mu\kappa\tau\nu} J^{\kappa\tau} P^\nu \quad , \tag{12}$$

with $J^{\kappa\tau}$ defined by Eqs. (9,10). The 4-momentum opertor P^ν can be replaced by its eigenvalue when acting on plane-wave eigenstates. Then we use the Lagrangian (3) and choose space-like normalized vector $n^\mu n_\mu = -1$ (for example $n_0 = 0$, $\mathbf{n} = \widehat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$). After lengthy calculations in a spirit of [43, p.58,147] one can find the explicit form of the relativistic spin:[3]

$$(W_\mu \cdot n^\mu) = -(\mathbf{W} \cdot \mathbf{n}) = -\frac{1}{2} \epsilon^{ijk} n^k J^{ij} p^0 \quad , \tag{13a}$$

$$\mathbf{J}^k = \frac{1}{2} \epsilon^{ijk} J^{ij} = \epsilon^{ijk} \int d^3\mathbf{x} \left[F^{0i} (\partial_\mu F^{\mu j}) + F_\mu{}^j (\partial^0 F^{\mu i} + \partial^\mu F^{i0} + \partial^i F^{0\mu}) \right] \quad . \tag{13b}$$

Now it becomes clear that application of the generalized Lorentz conditions (which are quantum versions of free-space dual Maxwell's equations) leads in such a formulation to the absence of electromagnetism in a conventional sense.[4] The resulting Kalb-Ramond field is longitudinal (helicity $h = 0$). The discussion of this fact can also be found in ref. [19,10].

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[3]Let me remind that the helicity operator is connected with the Pauli-Lyuban'sky vector in the following manner $(\mathbf{J} \cdot \widehat{\mathbf{p}}) = (\mathbf{W} \cdot \widehat{\mathbf{p}})/E_p$, see ref. [44].

[4]I would still like to point out one paradox connected with the presented treatment and the ordinary electrodynamics. On the basis of definitions:

$$\widetilde{F}^{\alpha\beta} = \frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} F_{\gamma\delta} \quad , \quad F^{\alpha\beta} = -\frac{1}{2} \epsilon^{\alpha\beta\gamma\delta} \widetilde{F}_{\gamma\delta} \tag{14}$$

and the use of the mathematical theorem that *any* antisymmetric tensor of the second rank can be expanded in potentials, *e.g.*, ref. [45] in the following manner:

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \quad , \quad \widetilde{F}_{\alpha\beta} = \partial_\alpha \tilde{A}_\beta - \partial_\beta \tilde{A}_\alpha \tag{15}$$

one can deduce the equivalent relations:

One of possible ways to obtain helicities $h = \pm 1$ is a modification of the electromagnetic field tensor like ref. [6q], *i.e.*, introducing the non-Abelian electrodynamics [7]:

$$F_{\mu\nu} \quad \Rightarrow \quad \mathbf{G}^a_{\mu\nu} = \partial_\mu A^{(a)\,*}_\nu - \partial_\nu A^{(a)\,*}_\mu - i\frac{e}{\hbar}[A^{(b)}_\mu, A^{(c)}_\nu] \quad , \tag{17}$$

where (a), (b), (c) are the vector components in the (1), (2), (3) circular basis [6,7]. In the other words, one can add some ghost field (the $B(3)$ field) to the antisymmetric tensor $F_{\mu\nu}$. As a matter of fact this induces hypotheses on a massive photon and/or an additional displacement current. We prefer to avoid any auxiliary constructions (even they are valuable in intuitive explanations and generalizations). If these constructions exist they should be deduced from a more general theory on the basis of some fundamental postulates, *e.g.*, in a spirit of refs. [27,46]. In the procedure of quantization one can reveal the important case, when transversality of the antisymmetric tensor field is preserved. This conclusion is connected with existence of the dual tensor $\tilde{F}^{\mu\nu}$ and with possibility of the Bargmann-Wightman-Wigner-type quantum field theory revealed in ref. [27]. The remarkable feature of the Ahluwalia *et al.* consideration is: boson and its antiboson can possess opposite relative parities.

We choose the field operator, Eq. (1), in the following way:

$$F^{i0}_{(+)}(\mathbf{p}) = E^i(\mathbf{p}) \quad , \quad F^{jk}_{(+)}(\mathbf{p}) = -\epsilon^{jkl}B^l(\mathbf{p}) \quad ; \tag{18a}$$

$$F^{i0}_{(-)}(\mathbf{p}) = \tilde{F}^{i0} = B^i(\mathbf{p}) \quad , \quad F^{jk}_{(-)}(\mathbf{p}) = \tilde{F}^{jk} = \epsilon^{jkl}E^l(\mathbf{p}) \quad , \tag{18b}$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the tensor dual to $F^{\mu\nu}$; and $\epsilon^{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}$, $\epsilon^{0123} = 1$ is the totally antisymmetric Levi-Civita tensor. After lengthy but standard calculations we achieve:

$$\begin{aligned}\mathbf{J}^k = \sum_{\eta\eta'}\int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p}\Bigg\{ &\frac{i\epsilon^{ijk}\mathbf{E}^i_\eta(\mathbf{p})\mathbf{B}^j_{\eta'}(\mathbf{p})}{2}\left[a_\eta(\mathbf{p})b^\dagger_{\eta'}(\mathbf{p}) + a_{\eta'}(\mathbf{p})b^\dagger_\eta(\mathbf{p}) + b^\dagger_{\eta'}(\mathbf{p})a_\eta(\mathbf{p}) + b^\dagger_\eta(\mathbf{p})a_{\eta'}(\mathbf{p})\right] - \\ &- \frac{i\mathbf{p}^k(\mathbf{E}_\eta(\mathbf{p})\cdot\mathbf{E}_{\eta'}(\mathbf{p}) + \mathbf{B}_\eta(\mathbf{p})\cdot\mathbf{B}_{\eta'}) - i\mathbf{E}^k_{\eta'}(\mathbf{p})(\mathbf{p}\cdot\mathbf{E}_\eta(\mathbf{p})) - i\mathbf{B}^k_{\eta'}(\mathbf{p})(\mathbf{p}\cdot\mathbf{B}_\eta(\mathbf{p}))}{2E_p}\times \\ &\times\left[a_\eta(\mathbf{p})b^\dagger_{\eta'}(\mathbf{p}) + b^\dagger_\eta(\mathbf{p})a_{\eta'}(\mathbf{p})\right]\Bigg\}\end{aligned} \tag{19}$$

We should choose normalization conditions. For instance, one can use an analogy with classical electrodynamics (a photon is massless):

$$(\mathbf{E}_\eta(\mathbf{p})\cdot\mathbf{E}_{\eta'}(\mathbf{p}) + \mathbf{B}_\eta(\mathbf{p})\cdot\mathbf{B}_{\eta'}) = 2E_p\delta_{\eta\eta'} \quad , \tag{20a}$$

$$\mathbf{E}_\eta \times \mathbf{B}_{\eta'} = \mathbf{p}\delta_{\eta\eta'} - \mathbf{p}\delta_{\eta,-\eta'} \quad . \tag{20b}$$

These conditions imply that $\mathbf{E} \perp \mathbf{B} \perp \mathbf{p}$. Finally, we obtain

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$$\partial_\nu F_{\alpha\beta} + \partial_\alpha F_{\beta\nu} + \partial_\beta F_{\nu\alpha} = 0 \Leftrightarrow \epsilon^{\mu\nu\alpha\beta}\partial_\nu F_{\alpha\beta} = 0 \;\Leftrightarrow\; \partial_\nu \tilde{F}^{\mu\nu} = 0 \quad , \tag{16a}$$

$$\partial_\nu \tilde{F}_{\alpha\beta} + \partial_\alpha \tilde{F}_{\beta\nu} + \partial_\beta \tilde{F}_{\nu\alpha} = 0 \Leftrightarrow \epsilon^{\mu\nu\alpha\beta}\partial_\nu \tilde{F}_{\alpha\beta} = 0 \;\Leftrightarrow\; \partial_\nu F^{\mu\nu} = 0 \quad . \tag{16b}$$

These relations state that currents cannot be put in the Maxwell's equations. Therefore, something should be corrected in our understanding of the nature of dual second-rank antisymmetric tensors, 4-currents and/or in the mentioned theorem.

$$\mathbf{J}^k = -i\sum_\eta \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\mathbf{p}^k}{2E_p}\left[a_\eta b^\dagger_{-\eta} + b^\dagger_\eta a_{-\eta}\right] \quad . \tag{21}$$

If we want to describe states with definite helicity quantum number (photons) we should assume that $b^\dagger_\eta(\mathbf{p}) = ia^\dagger_\eta(\mathbf{p})$ what is reminiscent with Majorana-like theories [34,35,47].[5] One can take into account the prescription of the normal ordering and set up the commutation relations in the form:

$$\left[a_\eta(\mathbf{p}), a^\dagger_{\eta'}(\mathbf{k})\right]_- = (2\pi)^3\delta(\mathbf{p}-\mathbf{k})\delta_{\eta,-\eta'} \quad . \tag{22}$$

After acting the operator (21) on the physical states, *i.e.*, $a^\dagger_h(\mathbf{p})|0>$, we are convinced that antisymmetric tensor field can describe particles with transversal components (helicity is equal to ± 1). One can see that the origins of this conclusion are the possibility of different definitions of the field operator and the existence of 'antiparticle' for a particle described by antisymmetric tensor field. The latter statement is related with the Weinberg discussion of the connection between helicity and representations of the Lorentz group [5a]. Next, I would like to point out that the Proca-like equations for antisymmetric tensor field with *mass*, *e.g.*, Eq. (4) can possess tachyonic solutions, see for the discussion in ref. [9]. Therefore, in a massive case the states can be "partially" tachyonic ones. We then deal with the problem of the choice of normalization conditions which could permit us to describe both transversal and longitudinal modes of the $j = 1$ field.

In conclusion, we calculated the Pauli-Lyuban'sky vector of relativistic spin on the basis of the Nötherian symmetry method [40–42]. Let me remind that it is a part of the angular momentum vector, which is conserved as a consequence of rotational invariance. After explicit [19] (or implicit [21]) applications of the constraints (the generalized Lorentz condition) in the Minkowski space, the antisymmetric tensor field becomes longitudinal (helicity h is equal to zero). We proposed one of possible ways to resolve this contradiction with the Correspondence Principle in refs. [9–12]. Another hypothesis has been proposed by Evans [6,7,49], in which the third component of the Pauli-Lyuban'sky vector has been identified with the new $B(3)$ field of electromagnetism.[6] The present article continues these researches. The conclusion achieved is: the antisymmetric tensor field can describe both the Maxwellian $j = 1$ field and the Kalb-Ramond $j = 0$ field. Nevertheless, we still think that the physical nature of the $E = 0$ solution revealed in refs. [48,26], its connections with the Evans-Vigier $B(3)$ field, ref. [6,7], with Avdeev-Chizhov δ'- type transversal solutions [21b], which cannot be interpreted as relativistic particles, as well as with my concept of χ boundary functions, ref. [12] are not completely explained until now. Finally, while we do not have any intention to doubt theoretical results of the ordinary quantum electrodynamics (it deals mainly with $E = \pm p$ solutions), we are sure that questions put forth in this note (as well as in previous papers of both mine and other groups) should be explained properly.

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[5]Of course, an imaginary unit can be absorbed by the corresponding re-definition of negative-frequency solutions.

[6]See also the paper of Chubykalo and Smirnov-Rueda [13]. The paper on connections between the Chubykalo and Smirnov-Rueda 'action-at-a-distance' construct and the $B(3)$ theory is in progress (private communication from A. Chubykalo).

of the manuscript I received the paper of Professor M. W. Evans "The Photomagneton and Photon Helicity" [49], devoted to a consideration of the similar topics, but from different points. His patient elucidations of the Evans-Vigier $B(3)$ model and useful information are acknowledged. I am delighted by the quality of referee reports on the papers [9–12] from "Journal of Physics A". In fact, they helped me to learn many useful things. The writing of the present paper has been inspired by the book of M. Ancharov "Kak ptitza Garuda".

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